Appendix A – Analysis of Mike's formulation of RFI feedback

# / Student	Feedback text	What is being refuted (**C**laim)	Reconstructed refutation argument (**D**ata, **W**arrant)
F3.1 Adrian	*These are critical points, but what makes you think that the maximum and minimum values of this polynomial on [0,1] are achieved at these points? The points do not even depend on the interval! Do you mean that the maximum and minimum values on every interval [a,b] are the same? But this cannot be, because the polynomial is unbounded both above and below.*	**C:** For every closed bounded interval *I*, if *f'(x)* has no roots then *f(x)* does not obtain a maximum or minimum in *I*.	**D:** The cubic polynomial *f* and its critical points. **W:** Let *I* be an arbitrary closed bounded interval and suppose that the maximum and minimum of *f* are achieved at the critical points of *f*, then *f* is bounded by its values at its critical points. Since *I* was arbitrary it follows that *f* is bounded which is a contradiction because non-constant polynomials are unbounded.
F3.2 Bailey	*f(x)=x does not have roots of the derivative (even among real numbers!) but it does achieve its maximum and minimum values on [0,1].*	**C:** For every polynomial *f*, if *f'(x)* has no roots then *f(x)* does not obtain a maximum or minimum in *I*	**D:** *f(x)=x.* **W:** The derivative of *f* has no roots, but *f* does achieve a minimum or maximum in *[0,1]*, contradiction!
F3.3 Charlie	*Apparently you see some connection between the sign of f''(0) and extremal values. Here is a counterexample to this connection: Consider your function on the closed interval [0,10]. It has no local maxima, its 2nd derivative is positive on (0,10], and f(0)=0 is not a maximum, since, say f(2) = 5/3 > 0. Thus, according to your logic, the function does not achieve a maximum value on [0,10].*	**C:** The maximum of *f* in *I* is necessarily achieved in points where the sign of *f''* is not positive.	**D:** The polynomial f, its critical points, the sign of *f''*. **W:** Suppose the maximum of *f* in *I* is necessarily achieved in points where the sign of f'' is not positive. Consider *f* on the closed interval *[0,10]*. The second derivative of *f* in (0,10] is positive and *f* does not achieve its maximum in *[0,10]* at *0* because f*(2)>f(0).* Thus, *f* does not achieve a maximum in *[0,10],* in contradiction to EVT.
F3.4 Dylan	*Note that both values of x [in which f'(x)=0] are outside the interval [0,1]. Thus, according to your logic, the range of your function does not have the LUB (nor LLB) even over the reals. Contradiction?*	**C:** If *f* is a real-valued polynomial then the maximum and minimum of *f(x)* in *[0,1]* are achieved at the critical points of *f(x).*	**D:** The polynomial *f* , the interval *[0,1],* the roots of f'(x) are both outside of the interval *[0,1].* **W:** Applying the attributed warrant to *f(x)* as a real-valued function implies that the real-valued *f(x)* does not attain a maximum or minimum in *[0,1],* in contradiction to EVT.